Proceedings of IAM, V.1, N.1, 2012, pp.15-35

PARETO OPTIMIZATION OF NANOFLUID FALKNER-SKAN WEDGE FLOW USING GENETIC ALGORITHM BASED ON NEURAL NETWORK MODELING

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Abstract. The steady two-dimensional boundary layer flow of nanofluid past a static wedge is numerically investigated. Two metamodels based on the evolved group method of data handling (GMDH) type neural networks are then obtained for modeling of both pressure drop parameter (PDP) and heat transfer parameter (HTP) with respect to design variables of volume fraction and Falkner-Skan power law parameter in considered problem. Resultant polynomial neural networks are deployed to find a set of optimal solutions, well known as Pareto optimal solutions, using multi-objective genetic algorithms (GAs) (non-dominated sorting genetic algorithm, NSGAII). It is shown that some useful and important information involved in the performance of Falkner-Skan wedge flow can be discovered by Pareto based multi-objective optimization.

Keywords: nanofluid, Falkner-Skan, neural network, GMDH, Pareto optimization, genetic algorithm.

AMS Subject Classification: 58E17, 92B20

1. Introduction

Increasing demand of cooling rate enhancement in high performance integrated electronic systems with exceedingly small dimensions was brought into happen using new class of fluids termed as the nanofluids. Dispersing nanoparticles with higher magnitude of thermal conductivity than the base fluid in liquids like water, ethylene glycol (EG), oils, etc. result in significantly increased cooling performance [1]. Due to enormous amount of heat generated by recent electronics, finding an efficient cooling system is one of the most important problems in designing electronic components. There are several studies for convective heat transfer in the literature. Effect of different particle volume percentages and different Reynolds number on the heat transfer coefficient of deionized water with a dispersion of Cu particles with below 100nm diameter as sample nanofluid were considered in some studies [2-4]. They showed an increasing Nusselt number with increasing volume loading of Cu-water nanofluids and Reynolds number. According to Lai et al. [5], the heat transfer coefficient depends on the nanofluid volume fraction, Reynolds number, the base fluid thermal properties, temperature and the nanoparticle purity. Considerable amount of studies were carried out regarding thermal conductivity of nanofluids [6-8]. Wang et al. [9] considered alumina and cupric oxide with a variety of base-fluid and showed enhanced thermal conductivity. A maximum of 12% increase in the thermal conductivity is noted with alumina particles of a volume fraction of 3%. However, the viscosity on the other hand showed an increase of 20–30% for the same volume fraction. Eastman et al. [10], reported 40% enhancement in thermal conductivity of 0.3% copper nanoparticles of ethylene glycol nanofluids compared to base fluid.

Finding optimal values of volume fraction and Falkner-Skan power law parameter is, indeed, a multi-objective optimization problem. Both the skin friction coefficient and the local Nusselt number of the flow and heat transfer are important objective functions to be optimized simultaneously in such a real world complex multi-objective optimization problem. These objective functions are figured out from experiment or using time consuming process of computer fluid dynamic (CFD) approaches, which cannot be used in an iterative optimization task unless a simple but effective metamodel is constructed over the numerical or experimental data.

System identification techniques are applied in many fields to model and predict the behaviors of unknown and/or very complex systems based on given input–output data [11]. Soft computing methods [12] are considered strictly in solving complex non-linear system identification and control problems. Many studies have been carried out to use evolutionary methods as effective tools for system identification [13-15]. Among these methodologies, the group method of data handling (GMDH) algorithm is a self-organizing approach by which gradually more complicated models are generated based on the evaluation of their performances on a set of multi-input, single output data pairs (x_i, y_i) $(i = 1,2,..., M)$. The GMDH was first developed by Ivakhnenko [16] as a multivariate analysis method for complex systems modeling and identification. In this way, the GMDH was used to circumvent the difficulty of having a priori knowledge of mathematical model of the process being considered.

Genetic algorithms have been used in a feed forward GMDH type neural network for each neuron searching its optimal set of connection with the preceding layer [17,18]. In the former reference, the authors have proposed a hybrid genetic algorithm for a simplified GMDH type neural network in which the connection of neurons are restricted to adjacent layers. Such shortcoming has been recently removed by the work of some authors [19,20].

Optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. Basically, the optimization process is defined as finding a set of values for a vector of design variables so that it leads to an optimum value of an objective or cost function. There are many calculus-based methods including gradient approaches to find mostly local optimum solutions and these are comprehensively explored in [21]. Strong dependence of gradient methods on the initial guess can cause gradient based methods to find a local optimum rather than a global one. This difficulty has led to extensive use of heuristic optimization methods, particularly genetic algorithms (GAs). When there are several objectives of cost functions that should be optimized simultaneously, the problem is considered as multi-objective optimization problem. Therefore, there is no single optimal solution that is best with respect to all the objective functions. Instead, there is a set of optimal solutions, well known as Pareto optimal solutions [22], which distinguish significantly the inherent natures between single-objective and multi-objective optimization problems. The concept of a Pareto front in the space of objective functions in multi-objective optimization problems stand for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. Both the NSGA and MOGA are Pareto based approaches which use the non-dominated sorting procedure originally proposed by Goldberg [23]. The lack of elitism in these algorithms was a motivation for modification of that algorithm to NSGA-II [24] in which a direct elitist mechanism has been introduced to enhance the population diversity.

In the present paper, effect of volume fraction and Falkner-Skan power law parameter are considered on skin friction coefficient and the local Nusselt number. An optimized GMDH type neural network are trained to best prediction of objectives for different designing parameters values. Obtained polynomial models are used to find pareto front of the best possible combinations of maximum Nusselt number and minimum skin friction coefficient. The corresponding variations of design variables, Nusselt number and skin friction coefficient, known as the Pareto set, constitute some important design choices that can be effectively used for optimal heat transfer with lower pressure drop in electronic device cooling systems using nanofluids.

In this study, numerical solution of Navier-Stokes and energy equations were obtained using MATLAB bvp4c and RK4 and validated using data reported by Yacob et al. [25]. Then, GS-GMDH type neural network are used to obtain polynomial models for simulating of HTP and PDP with values of m and ϕ. The obtained simple polynomial models are then used in a Pareto based multi-objective optimization approach to find the best possible combinations of HTP and PDP, known as the Pareto front.

2. Problem formulation

Consider the problem of steady two-dimensional boundary layer flow with water-based nanofluids containing Cu as nanoparticles past a wedge as depicted in Figure 1. No slip condition is taken into account and nanofluid is assumed to be incompressible along with laminar flow. Applying the boundary layer approximations and using the nanofluid model proposed by Tiwari and Das [26], the conservation of mass, momentum, and energy equations for a nanofluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

with the boundary conditions

$$v=0, u=0 \quad \text{at y}=0, \tag{4}$$

$$u=u_e(x) \quad \text{as y}\to\infty,$$

$$T=T_W \quad \text{at y}=0, \tag{5}$$

$$T=T_\infty \quad \text{as y}\to\infty.$$

Here u and v are the velocity components along the x and y direction, respectively, μ_{nf} and ρ_{nf} are the viscosity of the nanofluid and the density of the nanofluid respectively, and α_{nf} is the thermal diffusivity of the nanofluid which are given by

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}, \quad \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s, \tag{6}$$

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}, \tag{7}$$

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s, \tag{8}$$

where ρ_f is the density of the base fluid, ρ_s is the density of the solid particle, μ_f is the viscosity of the base fluid and ϕ is the solid volume fraction of the nanofluid. k_{nf} is the effective thermal conductivity of the nanofluid, which are approximated by the Maxwelle-Garnetts model (Oztop and Abu-Nada [27]).

For a main stream with velocity u_e varying as x^m, $u_e(x)=U_\infty x^m$ where U_∞ and m are constants with $0<m<1$, the transformations

$$\psi=\left[\frac{2\nu_f x u_e(x)}{m+1}\right]^{1/2} f(\eta), \quad \eta=\left[\frac{(m+1)u_e(x)}{2\upsilon_f x}\right]^{1/2} y, \tag{9}$$

$$\theta(\eta)=(T-T_\infty)/(T_W-T_\infty) \tag{10}$$

reduce the governing equations to [25]

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)}f'''+ff''+\beta\left(1-f'^2\right)=0, \tag{11}$$

$$\frac{1}{\Pr}\frac{\frac{k_{nf}}{k_f}}{\left[1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]}\theta''+f\theta'=0 \tag{12}$$

subject to the boundary conditions

$$f(0)=0, \quad f'(0)=0, \quad f'(\infty)=1, \tag{13}$$

$$\theta(0)=1, \quad \theta(\infty)=0, \tag{14}$$

where primes denote differentiation with respect to η, $\Pr = \nu_f / \alpha_f$ is the Prandtl number and the parameter β is the Hartree pressure gradient parameter which corresponds to $\beta = \Omega / \pi$ for a total angle Ω of the wedge defined as

$$\beta = \frac{2m}{m+1}. \tag{15}$$

The thermophysical properties of the fluid and nanoparticles are given in Table 1 (see Oztop and Abu-Nada [27]).

The skin friction coefficient C_f is defined as

$$C_f = \frac{\tau_W}{\rho_f u_e^2}, \tag{16}$$

with τ_W as the surface shear stress which is given by

$$\tau_W = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{17}$$

Substituting Eq. (9) into Eqs. (16) and (17) we obtain

$$PDP = \left[2\,\mathrm{Re}_x/(m+1)\right]^{1/2} C_f = \frac{1}{(1-\phi)^{2.5}} f''(0), \tag{18}$$

where $Re_x = u_e x / \nu_f$ is the local Reynolds number and PDP stands for pressure drop parameter that shows pressure drop over a wedge in Falkner-Skan flow quantitatively.

The local Nusselt number Nu_x is defined as

$$Nu_x = \frac{x q_W}{k_f (T_W - T_\infty)} \tag{19}$$

in which q_w is the surface heat flux which is designed as

$$q_W = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{20}$$

Using Eqs. (9), (19) and (20), we have

$$HTP = \left[(m+1)\mathrm{Re}_x/2\right]^{-1/2} Nu_x = -\frac{k_{nf}}{k_f}\theta'(0). \tag{21}$$

Here HTP stands for heat transfer parameter that shows heat transfer over a wedge in Falkner-Skan flow.

For the simulation of nanofluid flow field, Eqs. (11) and (12) along with boundary conditions (13) and (14) are solved numerically using two methods of

MATLAB bvp4c function and RK4. Further, numerical results are compared with those reported in the literature [25]. The numerical values of PDP and HTP for different values of m and ϕ are presented in Tables 2 and 3, respectively.

3. Modeling using GMDH type neural networks

Neural networks are composed of a number of components, named neuron, which are inspired by nature. These neurons construct different hidden layers in a neural network and connections between components make the network function. In GMDH type neural network, different pairs of neurons are connected together using a quadratic polynomial and make a neuron in next hidden layer. Such neural networks are adjusted so that a particular input leads to a specific target output. General function defining output with respect to input variables in a GMDH type neural network has the form of volterra functional series which is known as the Kolmogorov–Gabor polynomial [28,29] as in

$$f = a_0 + \sum_1^n a_i x_i + \sum_1^n \sum_1^n a_{ij} x_i x_j + \sum_1^n \sum_1^n \sum_1^n a_{ijk} x_i x_j x_k + \ldots \tag{22}$$

Eq. (22) can be constructed using different quadratic polynomials in the form of

$$\hat{y} = G\left(x_i, x_j\right) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2 \tag{23}$$

which is consisted of two neurons to make a neuron in next hidden layer.

Coefficients a_i in Eq. (23) should be calculated to perform minimum difference between neural network approximated value of output, $\hat{y}$, and its actual value, y, for each pair of (x_i, x_j) as the input variables. These coefficients are calculated using regression technique in a way that minimum difference between $\hat{y}$ and y is obtained for all M of input-output data in a least square sense, that is

$$E = \frac{\sum_{i=1}^{M} \left(y_i - \hat{y}_i\right)^2}{M} \to \min. \tag{24}$$

Singular value decomposition (SVD) is a popular technique to solve least square problems in existence of some singularities in the normal equations [20]. Using proposed technique in ref. [30, 31], the optimum values of a_i are calculated. According to ref. [32], self organizing GMDH type neural network performance is increased using such a technique of SVD. According to ref. [20], it seems that such application of SVD may remove the problem of superfluous data reported in ref. [33].

Genetic algorithm is used to find optimum network architecture between possible topologies [34]. Chromosomes are represented as proposed in ref. [35] in a way that neurons in different layers can be connected to neurons in layers far away. A sample GS-GMDH neural network is depicted in Fig. 2 which unlike the CS-GMDH neural networks, neurons connections can happen between every different layers not necessarily successive ones. In Figure 2, neuron ad in the first hidden layer is connected to the output layer by directly going through the second hidden layer. Therefore, it is now very easy to notice that the name of output neuron (network's output) includes ad twice as abccadad. In other words, a virtual neuron named adad has been constructed in the second hidden layer and used with abcc in the same layer to make the output neuron abccadad as shown in Figure 2.

Crossover and mutation are deployed to evolve generations [23] and roulette wheel selection approach is used for choosing two parents producing two offspring and selecting dominant chromosomes to transport to next generation. Also, elitism is used to bring the best population to next generation.

Crossover operator is depicted for a selected individual in the Figures 3 and 4. It should be noted that crossover location should be chosen randomly from set of $\left\{2^1, 2^2, 2^3, \ldots, 2^{n_l+1}\right\}$ where n_l is the number of hidden layers of the chromosome with the smaller length [20]. Mutation operator simply implemented by changing values of some genes.

4. Methodology for parametric optimization

In a multi-objective optimization problem, multiple objectives are optimized simultaneously which, there does not necessarily exist a solution that is best with respect to all objectives. Therefore, there is a set of optimum solutions that may be best in one objective but worst in another. In general, it can be mathematically defined as Find the vector $X^* = \left[x_1^*, x_2^*, x_3^*, \ldots, x_n^*\right]^T$ to optimize

$$F(X) = [f_1(X), f_2(X), \ldots, f_k(X)]^T \tag{25}$$

subject to m inequality constraints

$$l_i(X) \leq 0, \quad i = 1 \text{ to } m \tag{26}$$

and p equality constraints

$$h_j(X) = 0, \quad j = 1 \text{ to } p, \tag{27}$$

where $X^* \in \Re^n$ is the vector of decision or design variables, and $F(X) \in \Re^k$ is the vector of objective functions.

In implementing genetic algorithm, each chromosome is represented in binary string. The genetic operators of crossover and mutation are implemented to produce two offspring from two parents. The natural roulette wheel selection method is used for choosing two parents producing two offspring. The crossover operator for two selected individuals is simply accomplished by exchanging the tails of two chromosomes from a randomly chosen point. A simple mutation is performed by inverting the value of each gene with a small probability. Defining all objective functions in a way to be minimized, such multi-objective minimization based on the Pareto approach can be conducted using some definitions [20]:

4.1. Definition of Pareto dominance.

A vector $\mathbf{U} = [u_1, u_2, \ldots, u_k] \in \Re^k$ dominates to vector $\mathbf{V} = [v_1, v_2, \ldots, v_k] \in \Re^k$ (denoted by $\mathbf{U} < \mathbf{V}$) if and only

$$\forall i \in \{1, 2, 3, \ldots, k\}, u_i < v_i \Lambda \exists j \in \{1, 2, 3, \ldots, k\} : u_j < v_j \ .$$

It means that there is at least one u_j which is smaller than v_j whilst the rest u's are either smaller or equal to corresponding v's.

4.2. Definition of Pareto optimality.

A point $X^* \in \Omega$ (Ω is a feasible region in $\Re^n$ satisfying Eqs. (26) and (27)) is said to be Pareto optimal (minimal) with respect to all $X \in \Omega$ if and only if $F\left(X^{|*}\right) < F(X)$. Alternatively, it can be readily restated as

$$\forall i \in \{1,2,3,...,k\} \qquad \forall X \in \Omega - \{X^*\} f_i(X^*) <$$
$$< f_i(X) \Lambda \exists j \in \{1,2,3,...,k\} : f_j(X^*) < f_j(X).$$

It means that the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

4.3. Definition of Pareto set.

For a given multi-objective problem, a Pareto set $\wp^*$ is a set in the decision variable space consisting of all the Pareto optimal vectors $\wp^* = \left\{ X \in \Omega \middle| \nexists X' \in \Omega \ F(X') < F(X) \right\}$. In other words, there is no other X' in Ω that dominates any $X \in \Omega$.

4.4. Definition of Pareto front.

For a given multi-objective problem, the Pareto front $\wp \overline{T}^*$ is a set of vectors of objective functions which are obtained using the vectors of decision variables in the Pareto set $\wp^*$, that is

$$\wp \overline{T}^* = \left\{ F(X) = \left(f_1(X), f_2(X), \ldots, f_k(X) \right) : X \in \wp^* \right\}$$

Therefore, the Pareto front $\wp \overline{T}^*$ is a set of the vectors of objective functions mapped from $\wp^*$.

5. Modeling of HTP and PDP using GMDH-type neural network

HTP and PDP are considered as outputs which are dependent on input variables of are Falkner-Skan power law parameter and volume fraction of nanoparticle. Due to simultaneous solution of Eqs. (11) and (12) to find HTP and

PDP, equal number of input-output data pairs are considered to train GMDH-type neural network for both outputs. In order to demonstrate the prediction ability of the evolved GMDH type neural networks, the data in both input–output data have been divided into two different sets, namely, training and testing sets.

The training set consists of 80 out of the 100 input–output data pairs for HTP and alike for PDP which is used for training the neural network models using the method presented in Section 5. Remained 20 input-output data are then evaluated using trained GMDH-type neural network to convince ability to predict outputs. Genetic algorithm is used to find optimum network architecture to best training set data prediction. A population of 40 individuals with a crossover probability (P_c) of 0.7 and mutation probability (P_m) 0.08 has been used in 300 generations for both outputs. The corresponding polynomial representation for HTP is as follows:

$$y_{HL1N1} = 0.9139 + 2.5444\,\phi + 0.4808\,m - 0.492\,\phi^2 - \\ - 0.2819\,m^2 + 0.3014\,\phi\,m. \tag{28}$$

$$y_{HL2N1} = -1.4064 - 7.2316\,\phi + 3.7282\,y_{HL1N1} - \\ 9.3178\,\phi^2 - 1.3179\,{y_{HL1N1}}^2 + 7.001\,\phi\,y_{HL1N1}. \tag{29}$$

$$y_{HL2N2} = 0.9139 + 2.5444\,\phi + 0.4808\,m - 0.492\,\phi^2 - \\ - 0.2819\,m^2 + 0.3014\,\phi\,m. \tag{30}$$

$$y_{HL3N1} = 0.0076 + 0.9885\,y_{HL2N1} - 0.0234\,m + \\ + 0.009\,{y_{HL2N1}}^2 + 0.0537\,m^2 - 0.0217\,y_{HL2N1}\,m. \tag{31}$$

$$y_{HL3N2} = 0.0145 + 0.9677\,y_{HL2N2} + 0.0225\,m + \\ + 0.0164\,{y_{HL2N2}}^2 + 0.004\,m^2 - 0.0202\,y_{HL2N2}\,m. \tag{32}$$

$$\left[(m+1)\,\mathrm{Re}_x/2\right]^{-1/2} Nu_x = 0.0021 + 3.6673\,y_{HL3N1} - \\ 2.6695\,y_{HL3N2} - 0.4981\,{y_{HL3N1}}^2 + \\ + 0.5041\,{y_{HL3N2}}^2 - 0.0055\,y_{HL3N1}\,y_{HL3N2}. \tag{33}$$

Similarly, the corresponding polynomial representation of the model for PDP is in the form of

$$y_{HL1N1} = 0.4837 + 3.0694\,\phi + 1.6873\,m + \\ 3.7034\,\phi^2 - 1.0170\,m^2 + 3.5179\,m\,\phi. \tag{34}$$

$$y_{HL1N2} = 0.4837 + 3.0694\,\phi + 1.6873\,m + \\ 3.7034\,\phi^2 - 1.0170\,m^2 + 3.5179\,m\,\phi. \tag{35}$$

$$y_{HL2N1} = 0.0102 + 0.0825\,y_{HL1N2} + 0.9479\,\phi + \\ 0.0906\,{y_{HL1N2}}^2 + 0.0391\,\phi^2 - 0.1228\,y_{HL1N2}\,\phi. \tag{36}$$

$$y_{HL3N1} = -0.0897 + 1.2873\,m - 1.9522\,y_{HL1N1} - 0.1901\,m^2 - 7.7463\,{y_{HL2N1}}^2 + 2.5099\,m\,y_{HL2N1}. \tag{37}$$

$$y_{HL3N2} = 0.0003 + 1.0021\,y_{HL2N1} - 0.1057\,m + 0.0213\,{y_{HL2N1}}^2 + 0.277\,m^2 - 0.1168\,y_{HL2N1}\,m. \tag{38}$$

$$\left[2\,\mathrm{Re}_x/(m+1)\right]^{1/2} C_f = -0.0005 - 1.7404\,y_{HL3N1} + 2.7436\,y_{HL3N2} + 0.1965\,{y_{HL3N1}}^2 - 0.235\,{y_{HL3N2}}^2 + 0.0372\,y_{HL3N1}\,y_{HL3N2}. \tag{39}$$

The structures of the evolved three hidden layer GMDH type neural network for HTP and PDP are shown in the Figures 5 and 6 for HTP and PDP, respectively. As depicted in the Figures 7 and 8, the evolved GMDH-type neural network successfully model HTP and PDP, respectively. It is evident that this metamodel in terms of simple polynomial equations predict the outputs of the testing data that have not been used during the training process.

Obtained metamodels can now be utilized in a Pareto multi-objective optimization of the Falkner-Skan wedge flow considering both HTP and PDP as conflicting objectives.

6. Pareto optimization of Falkner-Skan wedge flow using polynomial neural network models

In order to investigate the optimal performance of the Falkner-Skan wedge flow in different values of ϕ and m, the metamodel obtained in the previous section are now deployed in a multi-objective optimization procedure. Two objective functions of HTP and PDP corresponding to design parameters of m and ϕ are easily evaluated using trained polynomial neural network which is obtained in pervious section. Since both HTP and PDP are maximized due to increasing the value of m and ϕ, while design goal is maximizing of HTP and decreasing in amount of PDP, therefore using multi-objective optimization algorithm is inevitable. Multi-objective optimization process results in non-dominated design points of input variables which are named pareto set. Non-domination means that improvement of an objective follow with worse value for other objectives.

Optimization is a time consuming process if objective values corresponding to input variables takes long to compute. Therefore, deploying a well trained neural network results in less time of optimization. Due to conflict of objective functions, it is not possible to find non-dominated pareto front without using of a multi-objective optimization algorithm.

Modified NSGA-II approach [19,20] are deployed in multi-objective optimization process where a population size of 60 has been chosen in different runs with crossover probability P_c and mutation probability P_m are 0.7 and 0.07, respectively. The range of variations for ϕ and m are assumed to be 0-0.2 and 0–1, respectively. Consequently, a total number of 71 non-dominated optimum design points have been obtained, as shown in Figure 9 in the plane of the HTP and PDP. Clearly, there are some important optimal design facts between the two objective functions that have been discovered by the Pareto optimization of the polynomial neural network models obtained using the numerical data of the Falkner-Skan wedge flow. There are three optimum design points, namely, a, b and c, whose corresponding design variables and objective functions are shown in Table 4. These points clearly demonstrate tradeoffs in objective functions HTP and PDP from which an appropriate design can be compromisingly chosen.

It can be readily seen from Figure 10 that for minimum value of m, value of HTP significantly increases from a to b with increase in value of ϕ, while less growth in HTP is obtained along with noteworthy increase in PDP corresponding to increment of m from b to c. Figures 11 and 12 depict the variations of ϕ and m on HTP. It can be readily seen that ϕ has more effect on variations of HTP. As can be seen from Figures 13 and 14, the value of PDP increases more as a result of increase in m value rather than ϕ. It is clear that such very useful informative design facts and tradeoffs have been only unveiled by using the Pareto multi-objective optimization approach of the simple polynomial neural network metamodels of numerical simulation of Falkner-Skan wedge flow.

7. Conclusions

Genetic algorithms have been successfully used both for optimal design of generalized GMDH type neural networks models of HTP and PDP of Falkner-Skan wedge flow and for multi-objective Pareto optimization of constructed metamodels. Two different polynomial relations for HTP and PDP have been found by evolved GS-GMDH type neural networks using some validated numerical simulations for input–output data of the Falkner-Skan wedge flow. The derived polynomial models have been then used in an evolutionary multi-objective Pareto optimization process so that some interesting and informative optimum design aspects have been revealed for Falkner-Skan wedge flow with respect to the its control variables of ϕ and m. Consequently, some very important tradeoffs have been obtained and proposed based on the Pareto front of two conflicting objective functions. Such combined application of GMDH type neural network modeling of input–output data and subsequent non-dominated Pareto optimization process of the obtained models is very promising in discovering useful and interesting design relationships.

Nomenclature			
C_f	skin friction coefficient	*Greek letters*	
C_p	specific heat at constant pressure	α	thermal diffusivity [$m^2\ s^{-1}$]
E	mean square of error	β	Hartree pressure gradient parameter
f	similarity function	ϕ	volume fraction of solid
H	auxiliary function	Φ	inverse mean square of error
k	thermal conductivity [$W\ m^{-1}K^{-1}$]	η	similarity variable
m	Falkner–Skan power law parameter	μ	Viscosity
Nu_x	local Nusselt number	θ	dimensionless temperature
P_c	crossover probability	ρ	Density [$Kg\ m^{-3}$]
P_m	mutation probability	Ω	total angle of the wedge
Pr	Prandtl number	τ	shear stress [$N\ m^{-2}$]
Q	heat flux	υ	kinematic viscosity [$m^{-2}s$]
Re	Reynolds number	ψ	stream function [s^{-1}]
T	Temperature [K]	*subscripts*	
U	free stream velocity [$m\ s^{-1}$]	f	base fluid
u, v	velocity components [$m\ s^{-1}$]	nf	nanofluids
$u_e(x)$	wedge flow free stream velocity	s	nano-solid particles
x, y	cartesian coordinates [m]	w	condition at the wall
		∞	ambient condition

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Table 1. Thermophysical properties of base fluid and nanoparticles

Physical properties	Fluid phase (water)	Cu
C_p (J/kg K)	4179	385
ρ (kg/m^3)	997.1	8933
k (W/mK)	0.613	400
$\alpha \times 10^{-7}$ (m^2/s)	1.47	1163.1

Table 2. The values of PDP for various values of *m* and ϕ

		$[2\ Re_x / (m+1)]^{1/2} C_f$		
ϕ	*m*	Nor Azizah Yacob [27]	bvp4c	Rk4
0.1	0	0.7179	0.7179	0.7179
0.2		0.9992	0.9992	0.9992
0.1	0.5	1.5881	1.5882	1.5882
0.2		2.2105	2.2106	2.2106
0.1	1	1.8843	1.8843	1.8843
0.2		2.6226	2.6227	2.6227

Table 3. The values of HTP for various values of *m* and ϕ

		$[(m+1)\ Re_x / 2]^{-1/2} Nu_x$		
ϕ	*m*	Nor Azizah Yacob [27]	bvp4c	Rk4
0.1	0	1.11	1.1101	1.1101
0.2		1.3342	1.3342	1.3342
0.1	0.5	1.3472	1.3473	1.3473
0.2		1.6048	1.6049	1.6049
0.1	1	1.4043	1.4043	1.4043
0.2		1.6692	1.6693	1.6693

Table 4. Design variables and objective functions values of significant Pareto points

Point	ϕ	*m*	HTP	PDP
A	0.0514	0.0515	1.0584	0.7517
B	0.1999	0.0647	1.4162	1.3157
C	0.1987	0.9805	1.6639	2.5859

Figure Caption

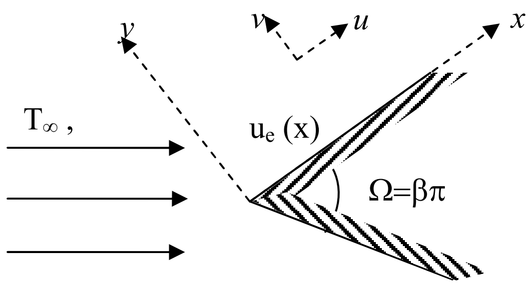

Figure 1

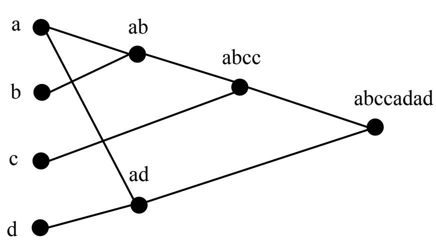

Figure 2

Parents		**Offsprings**
a b c c a d a d		a b c c
a b d d	→	a b d d a d a d

Figure 3

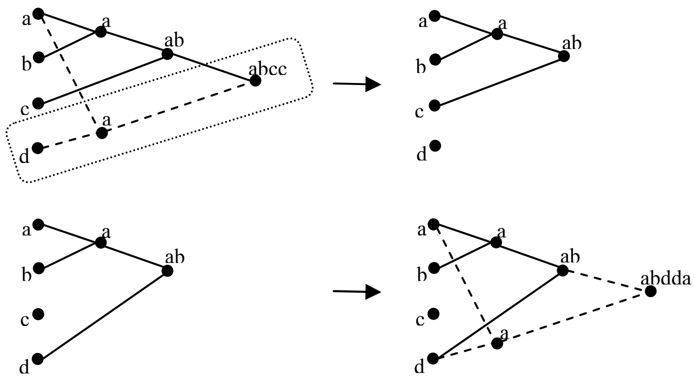

Figure 4

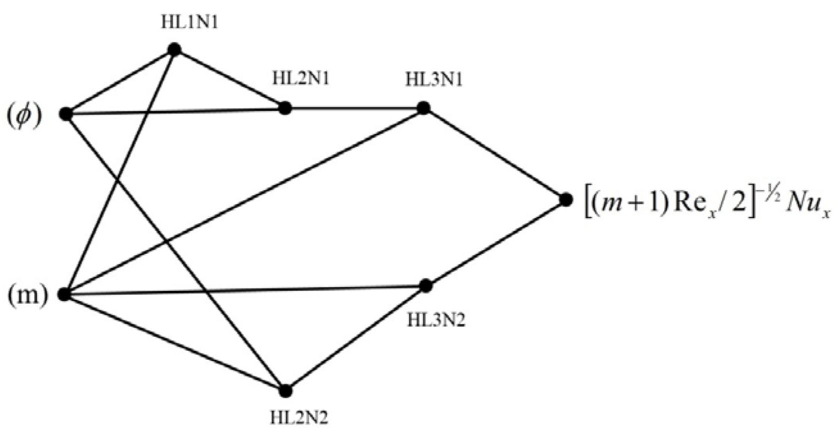

Figure 5

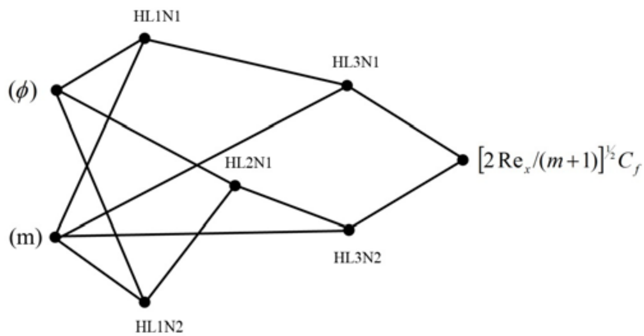

Figure 6

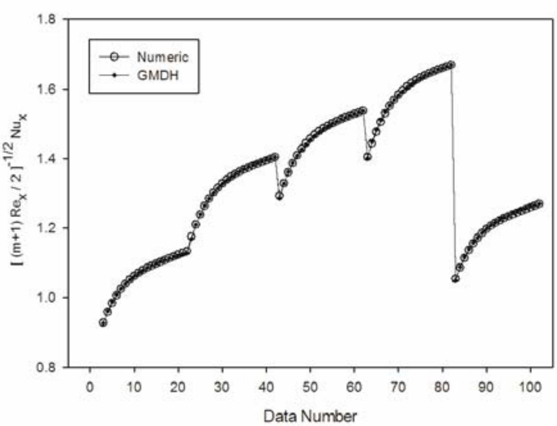

Figure 7

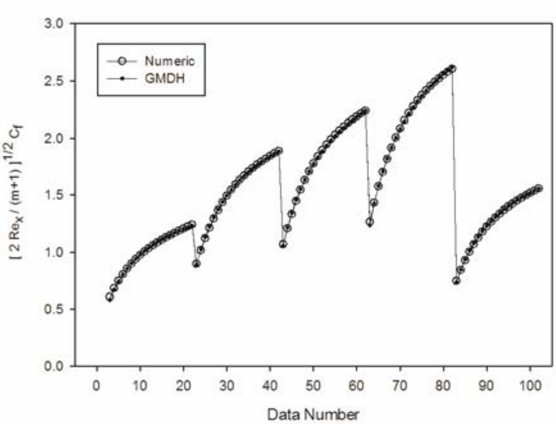

Figure 8

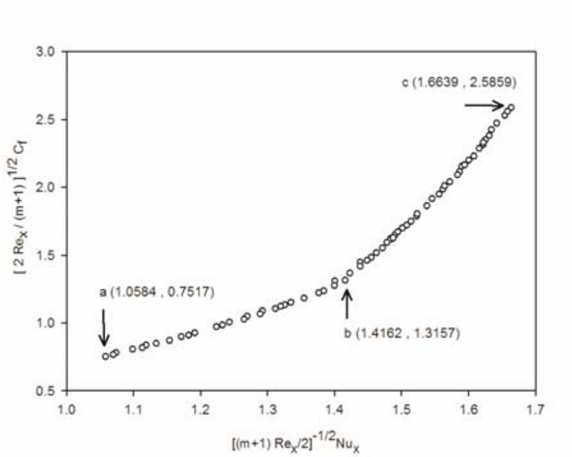

Figure 9

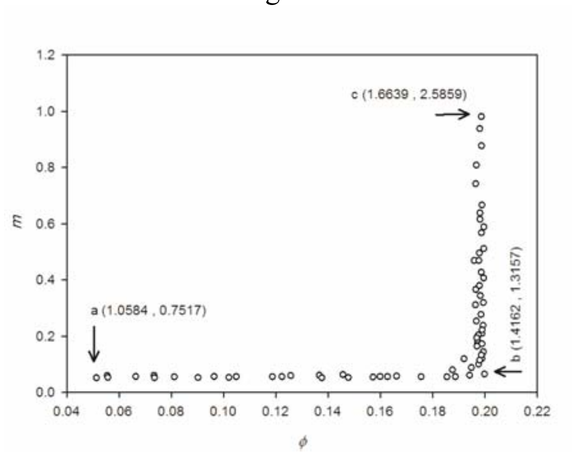

Figure 10

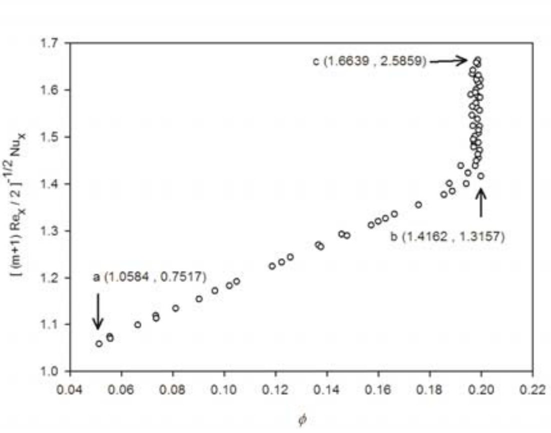

Figure 11

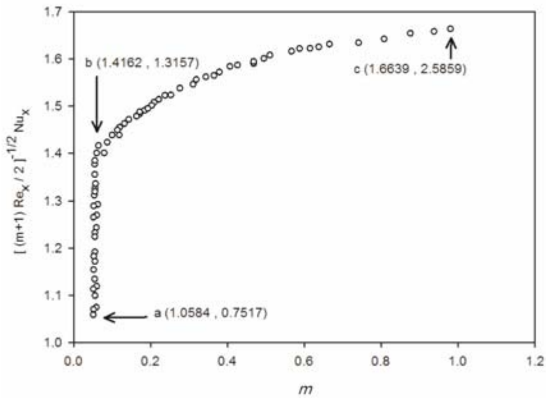

Figure 12

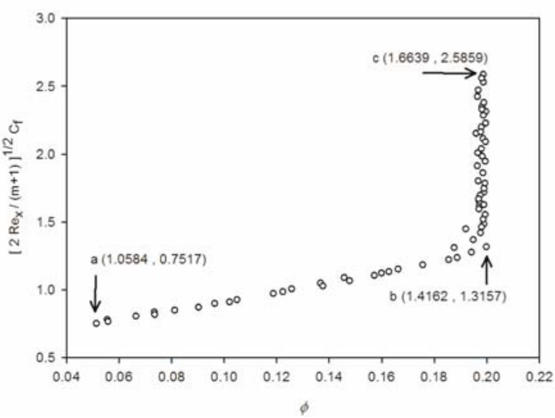

Figure 13

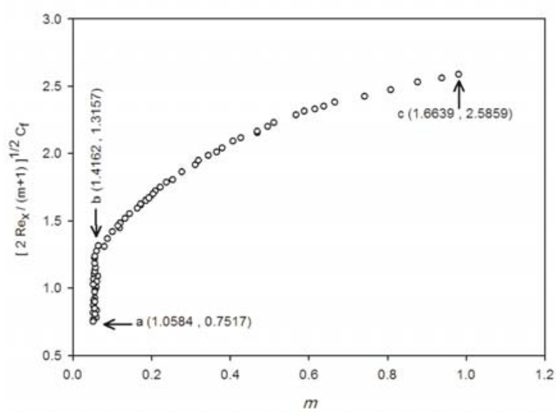

Figure 14

Nano-mayelərin paz-şəkilli Falkner-Skan axınının neyron şəbəkələrin optimallaşdırılmasına əsaslanan genetik alqoritmlərdən istifadə etməklə Pareto optimallaşdırılması

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XÜLASƏ

Nano mayenin paz-şəkilli axini zamanı yaranan sərhəd zolagının dayanıqlılıgı ədədi arashdırılır. Neyron şəbəkələr tipli verilənlərin işləmməsi üçün evolyusiya etmiş qrup metodu əsasında dizayn parametrlərinə nəzərən təzyiqin düşməsi, Folkner-Skan qanununun parametri ve istilik mübadiləsi parametrinin modellləşdirilməsi üçün iki metamodel təklif olunur. Alınmiş polinom neyron şəbəkələr çoxparametrli genetik algoritmlərdən (dominant olmayan seçim üçün genetik algoritmləri) istifadə etməklə Pareto optimal həllər adlanan optimal həllər çoxluğunun tapılmasi üçün istifadə olunur. Göstərilir ki, paz-shekilli axının Folkner-Skan çevirmələrində iştirak edən lazımlı informasiya çox kriteriyalı Pareto optimallaşdirma nəticəsində əldə edilə bilər.

Açar sözlər: nanofluid, Falkner-Skan, neyron şəbəkələr, Pareto optimallaşdırma, genetik alqoritm.

Парето оптимизация клинообразного патока типа Фалкнера-Скана нанофлюида используя генетического алгоритма основанного на моделирования нейронных сетей

А.П.М. Фаллах, А.Моради, Т.Хайят, А.А. Хенди

РЕЗЮМЕ

Численно исследуется устойчивый двумерный пограничный слой клинообразного потока статического нанофлюида. Получены два метамодели на основе эволюционированного групп метода для обработки данных типа нейронных сетей, для моделирования параметра падения давления и параметра теплообмена относительно переменных проектирования доля обмена и параметра закона Фолкнера-Скана для посмотренной задачи. Полученные полиномиальные нейронные сети использованы для нахождения множество оптимальных решений, известный как Парето оптимальных решений, с использованием многоцелевых генетических алгоритмов (генетические алгоритмы не доминируемой сортировки). Показано, что информация, содержащейся представлении Фолкнера-Скана клинообразного потока может быть обнаружена в результате многокритериальной Парето оптимизации.

Ключевые слова: нанофлюид, Фалкнер-Скан, нейронные сети, Парето оптимизация, генетический алгоритм.